DIVISIBILTY PROPERTIES OF GCD VE LCM MATRICES

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ABSTRACT

Let a, b and n be positive integers and let $S = \{x_1, x_2, \ldots, x_n\}$ be a set of distinct positive integers. The $n \times n$ matrix $(S_f) = (f((x_i, x_j)))$, having f evaluated at the greatest common divisor (x_i, x_j) of x_i and x_j as its $ij-$entry, is called the GCD matrix associated with f on the set S. Similarly, the $n \times n$ matrix $[S_f] = (f([x_i, x_j]))$ is called the LCM matrix associated with f on S. If $f = I$, the identity function on $\mathbb{Z}^+$, we have the classical GCD matrix (S) and the classical LCM matrix $[S]$. If $f = N^a$, the power function, we have the $a-$th power GCD matrix (S^a) and $a-$th the power LCM matrix $[S^a]$.

Let f be an integer valued arithmetical function. It is said that the matrix (S_f) divides the matrix $[S_f]$ in $M(n, \mathbb{Z})$ and denoted by $(S_f) \mid [S_f]$ if there exists an $n \times n$ matrix $B \in M(n, \mathbb{Z})$ such that $[S_f] = (S_f)B$. In [1], Bourque and Ligh showed that if S is factor closed then $(S)|[S]$. The set S is said to be factor closed if it contains every positive divisor of x for any $x \in S$. Hong [4] showed that such factorization is no longer true in general if S is gcd-closed. The set S is said to be gcd-closed if $(x_i, x_j) \in S$ for all $1 \le i, j \le n$. In this frame, many results on divisibility among GCD, LCM and related matrices are published in the literature. In this talk, we summarize these results and open problems.

Keywords. GCD matrix, LCM matrix, divisibility, gcd-closed set
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