

# On the $p$ -adic Birch and Swinnerton-Dyer conjecture

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**Abstract:** The celebrated conjecture Birch and Swinnerton-Dyer predicts the size of the abelian group of rational points on an elliptic curve  $E$  (called the Mordell-Weil group of  $E$ ) in terms of its Hasse-Weil  $L$ -function  $L(E, s)$ , a complex analytic object. In mid-80s Mazur, Tate and Teitelbaum formulated a  $p$ -adic version of this conjecture which seems more approachable via Iwasawa theoretic techniques. One then would like to compare the  $p$ -adic version to the original conjecture. I will report on a result where I proved that the  $p$ -adic analytic rank compares to the complex analytic rank exactly as predicted by Mazur-Tate-Teitelbaum. Along these lines, I will also report on the most recent results of Skinner and Venerucci relevant to this talk, who were able to prove the following statement: The Mordell-Weil group of  $E$  has rank one and the Tate-Shafarevich group is finite if and only if the entire function  $L(E, s)$  has a simple zero at  $s = 1$ .